Candidate: Leonardo Chirco Supervisor: Sandro Manservisi

# Development of numerical algorithms for optimal control problems in CFD

## Introduction

The aim of my research is to develop algorithms for optimal control problems related to many fields of CFD. In this work we considered a pressure boundary optimal control method for the Fluid Structure Interaction (FSI) problem based on Lagrangian multipliers and adjoint variables. We wrote the FSI problem in variational monolithic form and extended the velocity field in the solid region to balance automatically both the forces and the adjoint ones at the interface. The objective of the control is the matching of a displacement field in a particular region of the solid domain. This is accomplished by changing the pressure on the fluid inlet boundary which alters the fluid flow profile and thus deforms the shape.

## Mathematical model

A boundary optimal control is an *inverse* problem that consists in estimating one or more unknown boundary conditions of a system to achieve a desired objective. The ingredients of a FSI optimal control problem are

- **The fluid model.** We considered a Newtonian fluid described by the Navier-Stokes equations and an hyperelastic incompressible St.Venant-Kirchhoff material. The mathematical FSI model is then the following [a]

$$\begin{aligned} &\nabla\cdot\boldsymbol{v} = 0 && on \quad \Omega_f\,, \\ &\rho_f(\boldsymbol{v}\cdot\nabla)\boldsymbol{v} - \nabla\cdot\boldsymbol{T} = 0 && on \quad \Omega_f\,, \\ &\nabla\cdot\boldsymbol{S}(\boldsymbol{l}) = 0 && on \quad \Omega_s\,, \end{aligned}$$

  together with the appropriate boundary conditions.

- **The control parameter.** In this case the inlet boundary pressure.
- **The cost functional** to be minimized.

$$\mathcal{J}(\boldsymbol{l},p) = \frac{1}{2}\int_{\Omega_d} w(\boldsymbol{l}-\boldsymbol{l}_d)^2 d\Omega + \frac{1}{2}\int_{\Gamma_c} \beta\, p_c^2\, d\Gamma$$

The functional comprises the desired solid displacement $\boldsymbol{l}_d$, the control pressure $p_c$ and a Tikhonov regularization parameter $\beta$. To obtain the optimality system we write the full Lagrangian and impose the first order necessary conditions. The strong form of the adjoint system is recovered by setting to zero the Fréchet derivative taken with respect to the state variables and integrating some terms as well as the equation for the control pressure $p_c$

$$p_c = p = -\frac{\boldsymbol{v}_a\cdot\mathbf{n}}{\beta}\,.$$

Since the equations are strongly coupled and non linear we used an iterative steepest descent algorithm for the solution of the optimality system [b]. We implemented this algorithm in our in-house finite element code (**FEMuS**) which is parallelized with openMPI libraries and uses a multigrid solver with mesh-moving capability.

[a] D. Cerroni, L. Chirco, R. Da Vià, S. Manservisi, *Numerical simulation of a falling drop on a bending wall by coupling a VOF method with a Fluid Structure Interaction solver*, 15$^{th}$ ICNAAM Conference.
[b] L. Chirco, R. Da Vià, S. Manservisi, *An optimal control method for fluid structure interaction systems via adjoint boundary pressure*, 35$^{th}$ UIT Conference, Ancona, 2017

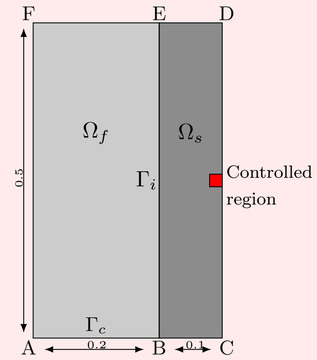
## Conclusions

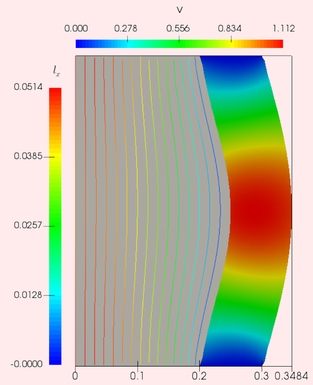
In this work we studied an optimal control method for the FSI problem. The results obtained with this new pressure boundary control method show great accuracy and robustness, this approach will be the starting point for the study of

- **More complex geometries**
- **Other functionals**
- **Velocity boundary control**

## Results

The domain used for the optimal control problem and the reference case without control are the following



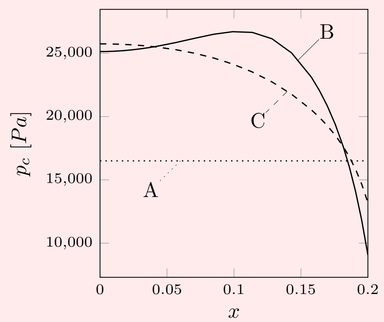



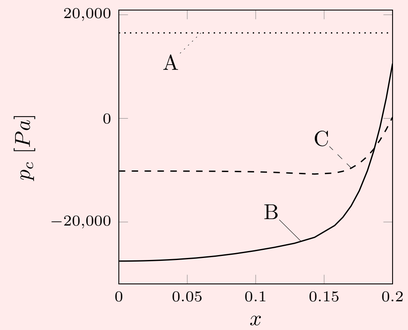

$AB$ is the fluid inlet, $EF$ the outlet and $BE$ the interface. The boundary conditions are $p_{EF} = 15000Pa$, $p_{AB} = 16500Pa$ for the reference case and $p_{AB} = p_c$ for the controlled one. The results refer to

(a) displacement enhancement case $\boldsymbol{l}_d = 0.07$.

(b) displacement reduction case $\boldsymbol{l}_d = 0.03$.

We performed many simulations, with $\beta$ ranging from $10^{-9}$ to $10^{-11}$, and reported the value of the functional for each case.




| | | A | | C | B |
|---|---|---|---|---|---|
| | $\beta$ | $\infty$ | $10^{-9}$ | $10^{-10}$ | $10^{-11}$ |
| Test (a) | $\mathcal{J}(\boldsymbol{l},p)$ | 9.30e-04 | 9.30e-04 | 1.16e-05 | 1.29e-07 |
| Test (b) | $\mathcal{J}(\boldsymbol{l},p)$ | 6.80e-04 | 7.59e-05 | 2.48e-05 | 1.71e-08 |



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